A Simple Space-Frequency Coding Scheme with Cyclic Delay Diversity for OFDM

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Abstract—*Cyclic Delay Diversity (CDD) is a simple approach to increase the frequency selectivity of the channel as seen by the receiver in an Orthogonal Frequency Division Multiplexing (OFDM) based transmission scheme. The reason for this is that CDD inserts virtual echos. Due to the virtual multipath components carriers experience different channels if the antenna specific cyclic delays are chosen properly. Without any additional effort the increased frequency selectivity can be exploited by using a Forward Error Correction (FEC) code, e.g., a convolutional code that benefits from an altered error distribution after demodulation.*

In this paper we investigate a simple multiple antenna transmission scheme that exploits the frequency selectivity of the channel even without FEC coding. Hereby, on each antenna a shifted version of the signal is sent which enables the receiver to apply optimum demodulation. We analyze the choice of the cyclic delay and present a comparison with the well-known Alamouti scheme. Additionally, we present simulation results for both schemes.

Index Terms— **space-frequency coding, transmit diversity, cyclic delay diversity**

I. INTRODUCTION

OFDM based transmission schemes lack on built-in diversity, which they suffer from. Additional methods have to be inserted to enable the exploitation of diversity in OFDM. In systems such as, e.g., DVB-T or HIPERLAN/2, solely interleaving in frequency direction is included that in fact helps to improve the performance in the case of frequency selective channels. In this case, for instance, a convolutional code can take advantage of the locally changed error density in the decoding process. But in channel environments that show typical flat fading characteristics there is no improvement due to this interleaving and thus, there are significant losses in terms of the Bit Error Rate (BER) and Frame Error Rate (FER). Therefore, there is a need to introduce some kind of diversity, e.g., spatial diversity, to OFDM based communication systems in order to satisfy the demand for high reliability and availability without increasing the transmit power or using additional bandwidth.

During the last few years a wide range of publications is devoted to Space-Time Codes (STCs) – in particular to their construction. Most of the investigations and analyzes are carried out for the particular case of a flat fading channel which is of special interest for the application in OFDM based transmission schemes. Some of these STCs achieve full spatial diversity and in some cases also additional coding gains. One well-known example of a STC that achieves full diversity but no coding gain is the Alamouti scheme [1] that originally was designed for two transmit antennas. Hereby, two consecutive symbols are processed in such a way in the transmitter that the signals on the antennas are orthogonal and can be easily combined in the receiver. This scheme can be extended to four antennas [2], [3], where three symbols are combined analogously. Furthermore, the simple construction rules of the Alamouti scheme allows for its application as a Space-Frequency Code (SFC) that also achieves full spatial diversity. Hence, in OFDM systems there is the choice of using the Alamouti scheme either as STC or as SFC.

In this paper we investigate a new approach to introduce spatial diversity to an OFDM transmission system. Thereby, we use CDD [4], [5] that inserts virtual echos and thus, increases the frequency selectivity of the channel seen by the receiver [6]. In order to exploit the inserted diversity even without using FEC coding or expensive combining schemes, we transmit on each antenna shifted versions of the signal, where this shifting is done in frequency direction and thus, we design a SFC. This transmission scheme can be easily transformed to a matrix-vector notation which is the basis for the optimum combining/demodulation. In addition, this new scheme is not restricted to a small number of antenna configurations.

This paper is organized as follows. In Section II, we first introduce the new transmission scheme, i.e., the cyclic shifting of the transmit symbols and the connection with CDD, and the receiver structure. Then, we show how the cyclic shifts have to be chosen in dependency of the modulation alphabet and the number of antennas in Section III.

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In Section IV, we present simulation results and a comparison with the Alamouti scheme for OFDM [7], [8]. Finally, we draw some conclusions in Section V.

II. Transmission Scheme

In Figure 1, the transmission scheme of the SFC with CDD is depicted.

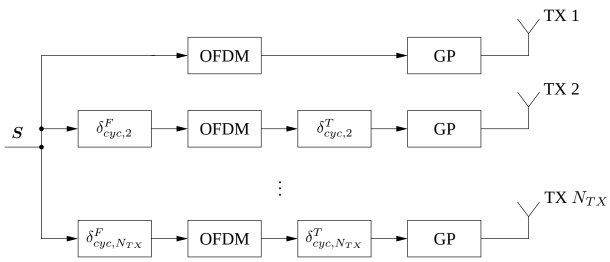

Figure 1. Transmission Scheme for N_{TX} Antennas

The signal vector $\boldsymbol{S} = (S_1\, S_2\, \ldots\, S_{N_{TX}})^T$ in the frequency domain is split to N_{TX} antenna branches and then cyclically delayed with an antenna specific shift $\delta^F_{cyc,n}, n = 1, 2, \ldots, N_{TX}$, according to Table 1 (we assume the shift of antenna one (TX 1) to be zero, i.e., $\delta^F_{cyc,1} = 0$, and thus, discard it in the following (confer Figure 1)). This corresponds to the SFC

$$\boldsymbol{S}_{SFC} = \begin{pmatrix} S_1 & S_2 & \ldots & S_{N_{TX}} \\ S_2 & S_3 & \ldots & S_1 \\ \vdots & \vdots & \ddots & \vdots \\ S_{N_{TX}} & S_1 & \ldots & S_{N_{TX}} \end{pmatrix}, \tag{1}$$

where each row is a space-frequency codeword and each column corresponds to a certain carrier. Thus, the number of transmit antennas N_{TX} is restricted by the number of carriers N_F, i.e., $N_{TX} \leq N_F$.

After that, the OFDM modulation is performed, i.e., the Inverse Fast Fourier Transform (IFFT), and then these signals are again cyclically delayed with the shift $\delta^T_{cyc,n}, n = 1, 2, \ldots, N_{TX}$, which corresponds to the CDD "encoding". The shifts are restricted to $0 \leq \delta^T_{cyc,n} \leq N_F - 1$, where N_F denotes the number of carriers. The prefix is added to fill the Guard Period (GP).

Carrier	TX 1	TX 2	$\cdots$	TX N_{TX}
f_1	S_1	S_2	$\cdots$	$S_{N_{TX}}$
f_2	S_2	S_3	$\cdots$	S_1
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$f_{N_{TX}}$	$S_{N_{TX}}$	S_1	$\cdots$	$S_{N_{TX}-1}$

Table 1. Transmission Scheme for N_{TX} Antennas in the Frequency Domain

The cyclic delays before and after the OFDM modulation (confer Figure 1) can both be performed before the OFDM modulation, i.e., in the frequency domain, and after the OFDM modulation, i.e., in the time domain, respectively. Here, we consider only the case, when both shifts are performed in the frequency domain. Then, the CDD signal in the frequency domain, as shown in equation (2), corresponds to the Phase Diversity (PD) signal.

$$\begin{aligned} s(l) &= \frac{1}{\sqrt{N_F}} \cdot \sum_{k=0}^{N_F-1} S(k) \cdot e^{j\frac{2\pi}{N_F}kl} \\ \underbrace{s((l-\delta_{cyc}) \bmod N_F)}_{\text{CDD signal}} &= \\ \frac{1}{\sqrt{N_F}} \cdot \sum_{k=0}^{N_F-1} & \underbrace{e^{-j\frac{2\pi}{N_F}k\cdot\delta_{cyc}} \cdot S(k)}_{\text{PD signal}} \cdot e^{j\frac{2\pi}{N_F}kl}. \end{aligned} \tag{2}$$

Using equation (2), the SFC $\boldsymbol{S}^{CDD}_{SFC}$ including CDD in the frequency domain yields

$$S^{CDD}_{SFC,k\,l} = S_{SFC,k\,l} \cdot e^{-j\frac{2\pi}{N_F}l\delta_{cyc,k}}. \tag{3}$$

The SFC without CDD after the Inverse OFDM modulation (IOFDM) can be transformed in a matrix-vector notation, where the channel matrix $\boldsymbol{H}$ results in a combined channel values/SFC matrix $\boldsymbol{H}_{SFC}$

$$\boldsymbol{R} = \boldsymbol{H}_{SFC} \cdot \boldsymbol{S} + \boldsymbol{N} \tag{4}$$

with $\boldsymbol{R} = (R_1\, R_2\, \ldots\, R_{N_{TX}})^T$, $\boldsymbol{S} = (S_1\, S_2\, \ldots\, S_{N_{TX}})^T$, the noise vector $\boldsymbol{N} = (N_1\, N_2\, \ldots\, N_{N_{TX}})^T$, and $\boldsymbol{H}_{SFC} = (H_{k\,l}),\ k, l \in [1, N_{TX}]$. Hereby, $H_{k\,l}$ is the channel of the symbol S_k from antenna TX l (we consider only one receive antenna). The CDD "encoding" results in a multipath channel seen at the receiver and thus, increases the frequency selectivity as depicted in Figure 2. The flat (black) plane is the channel at the receiver of the transmission without CDD. Using CDD some carriers experience a better channel and some a worth channel depending on the shift. The channel matrix can be denoted as:

$$\boldsymbol{H}^{CDD}_{SFC} = (H^{CDD}_{k\,l}),\ k, l \in [1, N_{TX}],$$

and thus

$$\boldsymbol{R} = \boldsymbol{H}^{CDD}_{SFC} \cdot \boldsymbol{S} + \boldsymbol{N}, \tag{5}$$

where now the total transmission scheme with cyclic shifts in the frequency and the time domain, respectively, and the channel are included in the matrix $\boldsymbol{H}^{CDD}_{SFC}$.

The optimum receiver has to evaluate the squared Euclidian distance in order to get an estimate $\hat{\boldsymbol{S}}$ of the transmit vector $\boldsymbol{S}$

$$\hat{\boldsymbol{S}} = \arg \min_{\boldsymbol{S} \in \mathcal{S}} \left\| \boldsymbol{R} - \boldsymbol{H}^{CDD}_{SFC} \boldsymbol{S} \right\|^2, \tag{6}$$

where $\mathcal{S}$ is the set of all signal vectors $\boldsymbol{S}$. Each transmit vector $\boldsymbol{S}$ corresponds to a vector $\boldsymbol{b} = (b_1\, b_2\, \ldots b_{q\cdot N_{TX}})^T$, when transmitting with an q-ary modulation alphabet. The Log-Likelihood Ratio (LLR) for bit $i, i = 1, 2, \ldots, q\cdot N_{TX}$, when receiving $\boldsymbol{R}$ is given by

$$\Lambda(b_i|\boldsymbol{R}) = \ln \left(\frac{\sum_{\boldsymbol{S} \in \mathcal{S}_i^{(0)}} e^{-\frac{1}{2\sigma_n^2}\left\|\boldsymbol{R} - \boldsymbol{H}^{CDD}_{SFC}\boldsymbol{S}\right\|^2}}{\sum_{\boldsymbol{S} \in \mathcal{S}_i^{(1)}} e^{-\frac{1}{2\sigma_n^2}\left\|\boldsymbol{R} - \boldsymbol{H}^{CDD}_{SFC}\boldsymbol{S}\right\|^2}} \right), \tag{7}$$

where $\mathcal{S}_i^{(0)}$ and $\mathcal{S}_i^{(1)}$ is the set of transmitted signals $\boldsymbol{S}$ with $b_i = 0$ and $b_i = 1$, respectively.

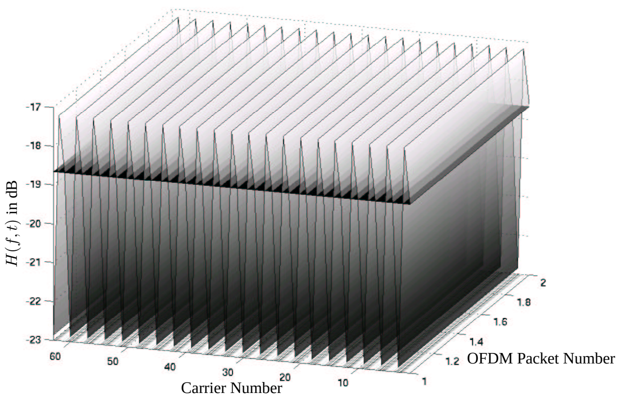

Figure 2. Channel Without and With CDD

III. CYCLIC DELAY ANALYSIS

In this section we investigate how the cyclic delay in the time domain $\delta_{cyc,n}^T, n = 1, 2, \ldots, N_{TX}$, has to be chosen in dependency of the modulation alphabet (here, we consider only Phase Shift Keying (PSK) modulation) and the number of transmit antennas to achieve best results. Hereby, we consider the complete SFC including CDD according to the encoding matrix of equation (3).

Example 1: Consider a two antennas SFC with CDD, where the cyclic shifts on the second antenna are $\delta_{cyc,2}^F = 1$ in the frequency domain and $\delta_{cyc,2}^T = N_F/2$ in the time domain. Then, the SFC from equation (3) is given by

$$\boldsymbol{S}_{SFC}^{CDD} = \begin{pmatrix} S_1 & S_2 \\ -S_2 & S_1 \end{pmatrix}$$

that is equivalent to the Alamouti scheme for two antennas and Binary PSK (BPSK) modulation and results in a real orthogonal 2×2 design.

In Example 1, it is shown that we can achieve full diversity and even an orthogonal design in the special case of BPSK transmission and two transmit antennas. In order to analyze the achievable diversity of the SFC with CDD we have to evaluate a modification for SFC of the diversity criterion for Rayleigh STCs [9], where we assume the channel to be flat over all carriers. Then, the matrix

$$\boldsymbol{B}(\boldsymbol{S}, \tilde{\boldsymbol{S}}) = \boldsymbol{S}_{SFC}^{CDD} - \tilde{\boldsymbol{S}}_{SFC}^{CDD} \qquad (8)$$

has to be full rank for $\boldsymbol{S}, \tilde{\boldsymbol{S}} \in \mathcal{S}, \boldsymbol{S} \neq \tilde{\boldsymbol{S}}$, to achieve maximum spatial diversity which is equal to the number of rows in the SFC matrix. Note, only a single receive antenna is considered.

In Table 2, all cyclic delays on the second antenna (TX 2) of a two antennas SFC with CDD are given that do not allow full diversity. For all other shifts the matrix in equation (8) has full rank and thus, the SFC achieves full spatial

Modulation Alphabet	$\delta_{cyc,2}^T$
BPSK	0
QPSK	0 / 32
8 PSK	0 / 16 / 32 / 48
16 PSK	0 / 8 / 16 / 24 / 32 / 40 / 48 / 56

Table 2. Cyclic Delay not Achieving Full Diversity for Two Antennas and $N_F = 64$ Carriers

diversity. Hereby, the number of carriers is $N_F = 64$ and the cyclic delays on the first antenna are $\delta_{cyc,1}^F = 0$ and $\delta_{cyc,1}^T = 0$. Further investigations for different numbers of carriers showed that the cyclic shifts for which full diversity cannot be achieved depend on the number of carriers N_F and the modulation alphabet $\mathcal{A}$ as follows:

$$\delta_{cyc,2}^T = i \cdot \frac{2N_F}{|\mathcal{A}|}, \quad i = 0, 1, \cdots \frac{|\mathcal{A}|}{2} - 1,$$

where $|\mathcal{A}|$ denotes the cardinality of the modulation alphabet, i.e., the number of signal points.

Modulation Alphabet	$\delta_{cyc,2}^T$	$\delta_{cyc,3}^T$
BPSK	15	31
QPSK	7	15
8 PSK	3	7

Table 3. One Set of Shifts Achieving Full Diversity for Three Antennas

Modulation Alphabet	$\delta_{cyc,2}^T$	$\delta_{cyc,3}^T$	$\delta_{cyc,4}^T$
BPSK	7	15	31
QPSK	7	15	31
8 PSK	3	7	15

Table 4. Example of a Set of Shifts Achieving Full Diversity for Four Antennas

In the Tables 3 and 4, one set of cyclic delays for different modulation alphabets is given for which full diversity can be achieved for the same parameters, i.e., $N_F = 64$ and $\delta_{cyc,1}^T = 0$. In all these schemes the shift in the frequency domain is $\delta_{cyc,i}^F = i - 1, i = 1, 2, \ldots, N_{TX}$. Of course, there are several such sets for which full diversity can be achieved. But compared to the two antennas case the number of possible choices of the shifts is strongly reduced and we are not able to derive a combination of the delays for an arbitrary number of carriers and modulation alphabet up to now.

In the next section we present simulation results for the BERs of the SFC with CDD and the Alamouti scheme as SFC [7], [8] for BPSK and QPSK modulation.

IV. SIMULATION RESULTS

In [7], [8], the Alamouti scheme for two antennas is applied to an OFDM based transmission scheme. Therefore, the transmission scheme is transformed in a matrix-vector notation [7]

$$\underbrace{\begin{pmatrix} R_1 \\ R_2^* \end{pmatrix}}_{\boldsymbol{R}} = \underbrace{\begin{pmatrix} H_1 & H_2 \\ H_2^* & -H_1^* \end{pmatrix}}_{\boldsymbol{H}} \cdot \underbrace{\begin{pmatrix} S_1 \\ S_2 \end{pmatrix}}_{\boldsymbol{S}} + \underbrace{\begin{pmatrix} N_1 \\ N_2 \end{pmatrix}}_{\boldsymbol{N}}, \qquad (9)$$

where $\boldsymbol{H}$ is a combined channel value/transmission scheme matrix. With this notation the soft demodulation is the same as in equation (7) and thus, optimum.

In Figure 3, the BERs of the one antenna and two antennas transmission schemes with the original Alamouti scheme as SFC respective new space-frequency coding scheme with CDD for BPSK modulation are shown. The cyclic shifts of the SFC with CDD are $\delta^F_{cyc,1} = 0$ and $\delta^F_{cyc,2} = 1$ respective $\delta^T_{cyc,1} = 0$ and $\delta^T_{cyc,2} = 32$. Additionally, the BER of the SFC with CDD with three and four antennas are given, where the shifts in the time domain are $\delta^T_{cyc,1} = 0$, $\delta^T_{cyc,2} = 15$, and $\delta^T_{cyc,3} = 31$ respective $\delta^T_{cyc,1} = 0$, $\delta^T_{cyc,1} = 7$, $\delta^T_{cyc,1} = 15$, and $\delta^T_{cyc,1} = 31$. The channel has typical flat fading characteristics. The two antennas SFC is identical with the Alamouti scheme and thus, the same performance is obtained, i.e., full spatial diversity. With the three and four antennas SFC with CDD additional gains can be reached.

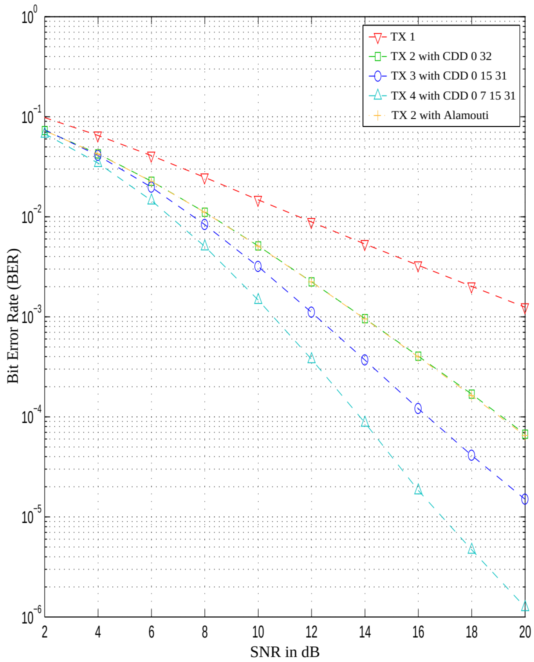

Figure 3. Comparison of the Alamouti Scheme and SFC with CDD for BPSK Modulation

In Figure 4, the equivalent BERs for QPSK modulation are shown. The shifts in the time domain are now $\delta^T_{cyc,2} = 16$ for the two antennas case and $\delta^T_{cyc,2} = 7$ and $\delta^T_{cyc,3} = 15$ for the three antennas case according to Table 3. Now, the two antennas SFC with CDD is constantly about 1.8 dB worse than the Alamouti scheme over the whole SNR range. This means, that we achieve full diversity (slope of the curve), but have a constant loss, since this scheme is not orthogonal and thus, there is additional interference as compared to the Alamouti scheme. With three antennas additional gains can be obtained in high SNR regions.

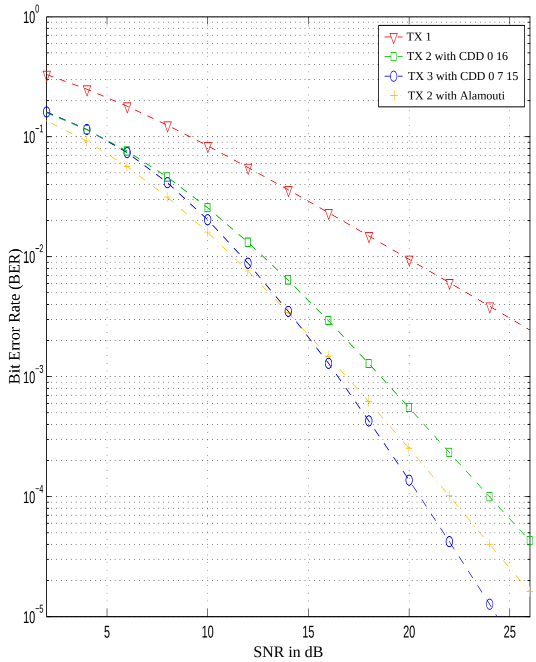

Figure 4. Comparison of the Alamouti Scheme and SFC with CDD for QPSK Modulation

The soft demodulation in equation (7) is necessary when a FEC code with a Soft Input (SI) decoding algorithm is used. In order to minimize the interferences of the SFC with CDD when transmitting uncoded optimum demodulation according to equation (6) is necessary. Therefore, it should be mentioned that the complexity of these kinds of demodulations increases with the modulation alphabet $\mathcal{A}$ and the number of transmit antennas N_{TX} with $|\mathcal{A}|^{N_{TX}}$. On the other hand, since the new transmission scheme allows the use of the optimum demodulation, the performance can be kept even if the interferences are increased by a channel with frequency selective characteristics, i.e., if the assumptions of a flat fading channel are not valid. In Figure 5, the performance of the SFC with CDD, the Alamouti scheme with optimum demodulation and adapted respective non-adapted channel values are shown for a transmission over a multipath channel, i.e., a highly frequency selective channel. The BPSK modulation alphabet is used.

The Alamouti scheme with the adapted channel values and the SFC with CDD perform as well as for a flat fading channel (both are the same and thus, only one curve is plotted). This means, that we can eliminate the interferences completely for the Alamouti scheme, even if the space-frequency codewords at the receiver are not longer orthogonal. In contrast to this the Alamouti scheme with non-adapted channel values shows a severe loss in the bit error performance. For the Alamouti scheme with four antennas and arbitrary modulation alphabet, it is in general not possible to apply the optimum demodulation. Therefore, the SFC with CDD is favorable for more than two transmit antennas and frequency selective channels.

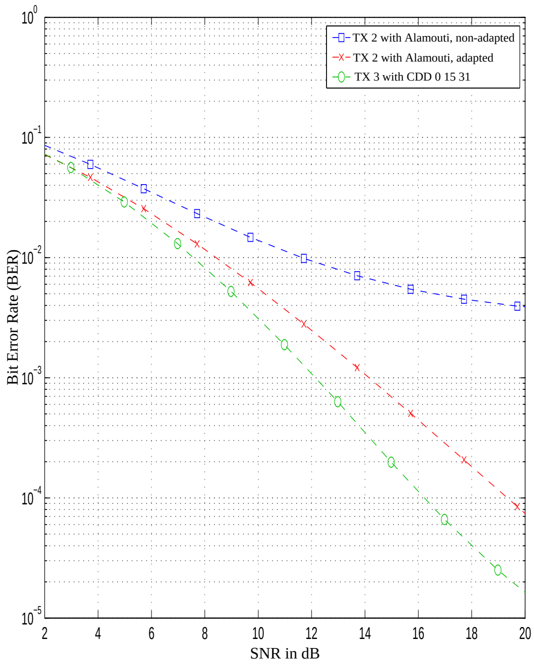

Figure 5. Comparison of the Alamouti Scheme and SFC with CDD for Transmission over a Frequency Selective Channel

V. Conclusions

In this paper we have investigated the performance of a new space-frequency coding scheme for OFDM based transmission schemes – namely SFC with CDD. Its encoding procedure is simple and there are only $2(N_{TX} - 1)$ shifts necessary, $N_{TX} - 1$ in the frequency domain and $N_{TX} - 1$ in the time domain. If the cyclic shifts in the time domain are chosen carefully, it has been shown that the encoding matrix has full rank and thus, full spatial diversity can be achieved. Thereby, the number of transmit antennas and the modulation alphabet (for PSK modulation) play an important role. In addition, the SFC with CDD is not restricted to a few special antenna configurations.

Furthermore, the SFC with CDD as well as the Alamouti scheme for two antennas performs very well, when the assumption of a flat channel over all carriers does not hold, due to the optimum demodulation. This may be an advantage of the new scheme proposed here, when considering more than two transmit antennas. For the optimum respective optimum soft demodulation needs a equivalent matrix-vector notation that does not exist for any SFC, but for the SFC with CDD.

Simulation results have shown that this encoding scheme is equivalent to the Alamouti scheme for two antennas and BPSK modulation for an appropriate choice of the cyclic delay in the time domain. For other constellations the BERs have shown that full spatial diversity can be reached and that the scheme is robust against deviations from the ideal assumptions.

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