On Cyclic Delay Diversity in OFDM Based Transmission Schemes

M. Bossert†, A. Huebner†, F. Schuehlein†, H. Haas‡, and E. Costa‡

† University of Ulm
Department of Telecommunications and
Applied Information Theory
Albert-Einstein-Allee 43
D-89081 Ulm, Germany
{martin.bossert, axel.huebner}@e-technik.uni-ulm.de

‡ Siemens AG
Information and Communication Mobile Networks
Future Radio Concepts
Gustav-Heinemann-Ring 115
D-81739 Munich, Germany
{harald-1.haas, elena.costa}@icn.siemens.de

***Abstract*— Cyclic Delay Diversity (CDD) is a simple approach to introduce spatial diversity to an Orthogonal Frequency Division Multiplexing (OFDM) based transmission scheme that itself has no built-in diversity. It also can be regarded as a Space-Time Code (STC). But in contrast to that there is no additional effort in the receiver necessary, since the different codewords result in a changed channel impulse response in the receiver. They insert virtual echos and thus increase the frequency selectivity of the channel seen by the receiver.**

In this paper we investigate CDD for uncoded and coded transmission and show how the inserted diversity can be exploited. Additionally, we analyze how to choose the cyclic shift at each transmit antenna to achieve best possible results, and present a comparison with the well-known Alamouti scheme.

***Index Terms*—cyclic delay diversity, space-time code**

I. Introduction

OFDM based transmission schemes suffer from the lack of built-in diversity. Therefore, it is necessary to introduce some kind of diversity, e.g., spatial diversity, to such a transmission system in order to achieve high reliability and availability without using additional bandwidth. In systems such as, e.g., DVB-T or HIPERLAN/2, merely interleaving in frequency direction is included, which only in the case of frequency selective channels can help to improve the decoding performance of the convolutional code. But for channels that show typical flat fading characteristics one cannot take advantage of the interleaving which results in significant losses in terms of the Bit Error Rate (BER) respective Frame Error Rate (FER).

At the same time, a wide range of publications is devoted to the investigation and construction of STCs during the last few years. Many of these analyzes are carried out for the special case of a flat fading channel which is very interesting for OFDM based transmission schemes. Some of the STCs achieve full diversity and possibly additional coding gains. One well-known example is the Alamouti scheme [1] that achieves full diversity. Hereby, two consecutive symbols are processed in such a way that they can be easily combined in the receiver.

In [2], a spatial diversity scheme for OFDM is introduced, namely transmit Delay Diversity (DD), where on the first antenna the non-delayed signal and on the second or each additional antenna delayed versions of the signal are transmitted. This scheme has a very simple receiver structure – the receiver remains unchanged compared with the single antenna case. But the main issue of this scheme is that the possible respective maximum delays are strongly restricted by the Guard Period (GP). Likewise, transmit Cyclic Delay Diversity (CDD) [3], [4] provides the same simple receiver structure. Hereby, the signal is not truly delayed but cyclically shifted and thus, there are no restrictions for the cyclic shift. An equivalent representation of CDD in the frequency domain is Phase Diversity (PD) [3] that provides the same properties. In this paper we only want to consider CDD, since this diversity method seems to be the most promising choice with respect to the transmitter and receiver structure.

This paper is organized as follows. In Section II, we first introduce CDD, i.e., the structure of a multiple antenna transmitter. Then, we show how the inserted diversity can be exploited, and analyze how the cyclic shifts have to be chosen in dependency of the modulation alphabet in Section III (we consider only Phase Shift Keying (PSK) modulation). In Section IV, we present simulation results and a comparison with the Alamouti scheme for two antennas [1]. Finally, we draw some conclusions in Section V.

Dr. Haas is now with the International University Bremen (IUB), Campus Ring 1, 28759 Bremen, Germany

II. TRANSMISSION SCHEME

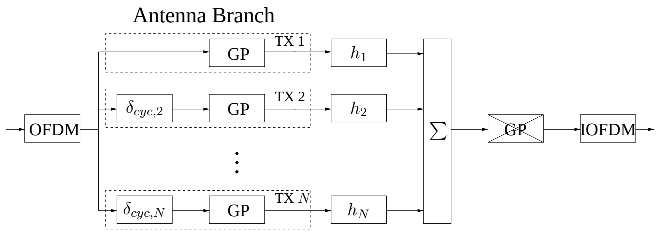

Fig. 1. The CDD Transmission Scheme

In Fig. 1, the transmission scheme of an N antennas transmit CDD system is shown. The signal is split after the OFDM modulation, i.e., the Inverse Fast Fourier Transform (IFFT), to N antenna branches. In each branch the OFDM symbol is cyclically shifted by an antenna specific delay $\delta_{cyc,n}, n = 1, 2, \ldots, N$. From the equivalent representation of CDD [3] in the frequency domain in equation (2) that corresponds to the PD signal, we can see that it only makes sense to choose the cyclic shift $\delta_{cyc,n}$ out of the interval $0 \leq \delta_{cyc,n} \leq N_F - 1$, where N_F denotes the number of carriers. Thus, the number of antennas is also restricted to the number of carriers N_F. Furthermore, we assume the delay of antenna one (TX 1) to be zero, i.e., $\delta_{cyc,1} = 0$, and discard it in the following (confer Fig. 1).

$$s(l) = \frac{1}{\sqrt{N_F}} \cdot \sum_{k=0}^{N_F-1} S(k) \cdot e^{j\frac{2\pi}{N_F}kl} \quad (1)$$

$$\underbrace{s((l-\delta_{cyc}) \bmod N_F)}_{\text{CDD signal}} = \quad (2)$$

$$\frac{1}{\sqrt{N_F}} \cdot \sum_{k=0}^{N_F-1} \underbrace{e^{-j\frac{2\pi}{N_F}k\cdot\delta_{cyc}} \cdot S(k)}_{\text{PD signal}} \cdot e^{j\frac{2\pi}{N_F}kl}.$$

The prefix is added to fill the GP. On the channel these signals superimpose and the receiver processes the sum signal by simply removing the GP and performing the Inverse OFDM (IOFDM) modulation. This is possible, since the cyclic shifts appear as multipaths at the receiver and thus, no special combining and no additional effort is necessary (except for the channel estimation).

These virtual echos change the characteristics of the channel in such a way that it seems to be frequency selective and thus, the error distribution of the uncoded transmission changes. Fig. 2 and 3 show the distribution of uncoded errors in the time-frequency-plane for transmission over a flat fading channel without CDD and with CDD, respectively. The number of carriers is $N_F = 64$ and the cyclic delay on the second antenna is $\delta_{cyc,2} = 2$ (in samples).

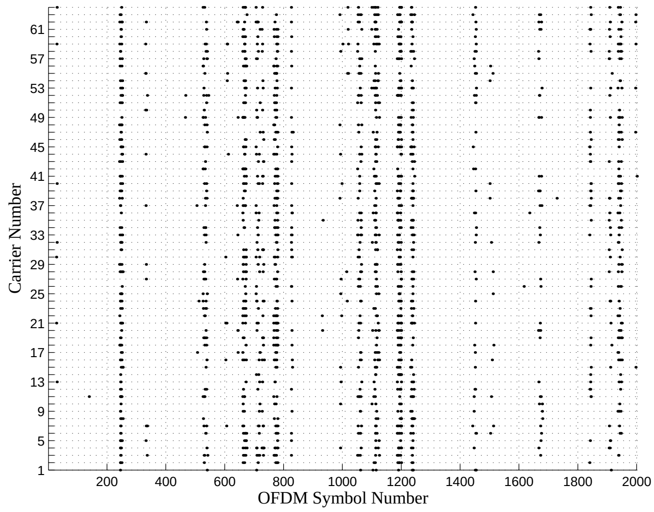

Fig. 2. Uncoded Error Distribution without CDD (• Error)

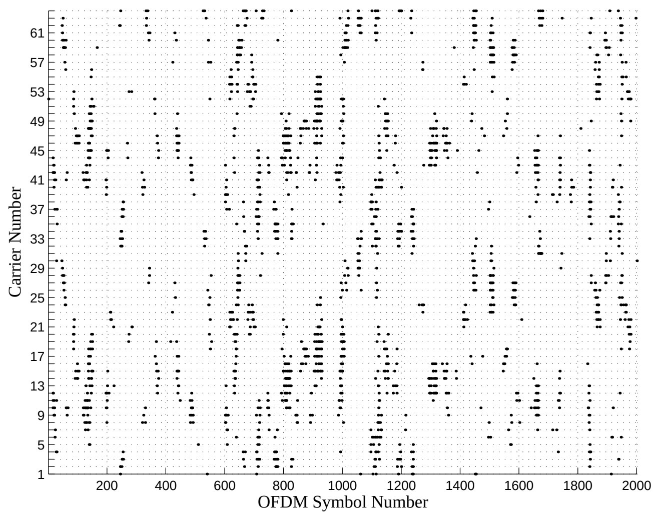

Fig. 3. Uncoded Error Distribution with CDD, $\delta_{cyc,2} = 2$ (• Error)

Compared to the case without CDD (Fig. 2) the error distribution changes if we apply CDD on the second antenna (see Fig. 3) – but not the total number of errors. This can be seen easily, when we consider the BERs of these two cases above. In Fig. 4, the uncoded respective coded BERs are depicted. The uncoded performance is in both cases identical, which means that the number of errors stay rather the same. But the error distribution changes and that is what the convolutional code can gain from. Hence, the coding gain is considerably larger in the CDD case.

In the next section we investigate how to chose the cyclic delay to get best results, on the one hand in dependency of the modulation alphabet and on the other hand if we use more than two transmit antennas.

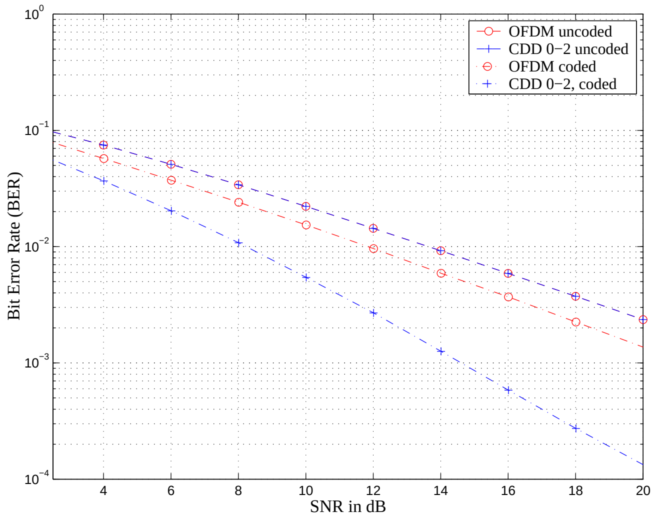

Fig. 4. Uncoded and Coded BER without and with CDD

III. Cyclic Shift Analysis

In the last section we have seen that we can gain from CDD when applying Forward Error Correction (FEC) coding. But how has the cyclic shift to be chosen to achieve best results? And how can this be extended to more than two transmit antennas? This can be analyzed using equation (2), where we can see that the shift corresponds to a phase rotation, which is important when considering different modulation alphabets.

In Fig. 5, the mean (flat plane) and the variance of the channel seen by the receiver for a transmission with two antennas and CDD over flat fading channels is depicted, where on both antennas the cyclic delay is varied in the range of $0 \leq \delta_{cyc,n} \leq N_F, n = 1, 2$. The mean of the channel does not change, but the variance that is completely symmetric. It has a maximum, when the difference of both shifts is maximum, i.e.,

$$\Delta_{opt}^{BPSK} = |\delta_{cyc,1} - \delta_{cyc,2}| = 32 = N_F/2.$$

Thus, the performance is only dependent on the relative shift of antenna one to antenna two and it is optimum for $N_F/2$ for BPSK modulation.

For more than two antennas the best combination of the cyclic delays is not so obvious. Simulations have shown that the mean is constant for any shift. But the maxima of the variance are not in high gear, as shown in Fig. 6, where the cyclic delay of the first antenna is constant zero, i.e., $\delta_{cyc,1} = 0$. It seems to be a good choice in this case to divide the possible range equidistantly, e.g., $\delta_{cyc,1} = 0$, $\delta_{cyc,2} = 21$, and $\delta_{cyc,3} = 42$.

For QPSK modulation and two antennas investigations have shown that the optimum cyclic delay on the second antenna is $\delta_{cyc,2} = 16$, where we used a shift of $\delta_{cyc,1} = 0$ on the first antenna. This means that the difference of both shifts for QPSK is now

$$\Delta_{opt}^{QPSK} = |\delta_{cyc,1} - \delta_{cyc,2}| = 16 = N_F/4,$$

i.e., half of the optimum shift for BPSK. Thus, the optimum cyclic shift seems to behave reciprocal to the cardinality of the modulation alphabet $\mathcal{A}$, i.e.,

$$\Delta_{opt}^{\mathcal{A}} = |\delta_{cyc,1} - \delta_{cyc,2}| = \frac{N_F}{|\mathcal{A}|}, \tag{3}$$

where we only consider PSK modulation.

The BERs for BPSK and QPSK modulation are shown in the next section, where we also compare it with the well-known Alamouti [1] scheme for two antennas.

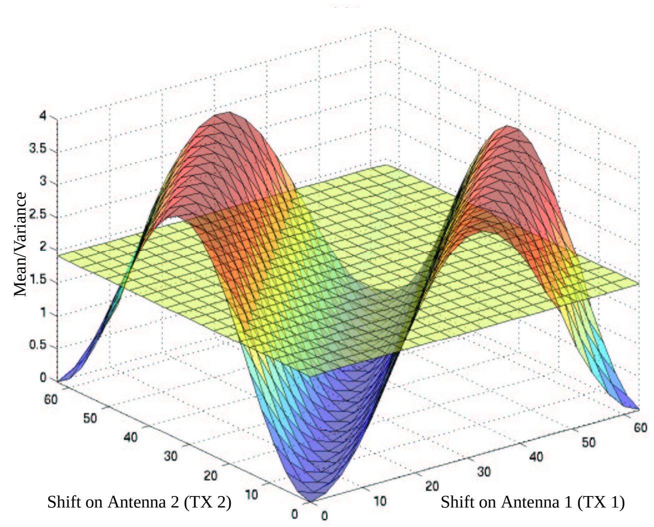

Fig. 5. Mean/Variance of the Channel with Two Transmit Antennas

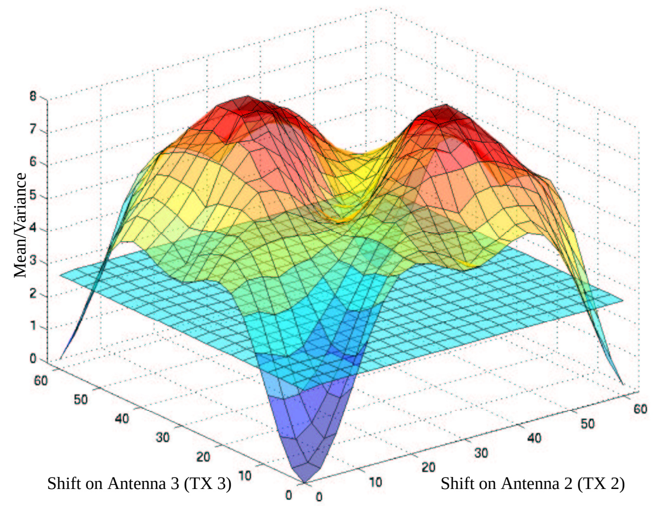

Fig. 6. Mean/Variance of the Channel with Three Transmit Antennas

IV. Simulation Results

In [1], a transmit diversity scheme for two antennas is proposed by Alamouti that achieves full spatial diversity. We call it the Alamouti scheme. Thereby, every two consecutive symbols are combined and sent in the way shown in Tab. I. In [5] and [6], it is applied to an OFDM based

	TX 1	TX 2
Time t	S_1	S_2
Time $t+T$	$-S_2{}^*$	S_1^*

TABLE I

Transmission Scheme for the Information Symbols S_1 and S_2

system. The scheme is transformed to a matrix-vector-notation [5]

$$\underbrace{\begin{pmatrix} R_1 \\ R_2^* \end{pmatrix}}_{\boldsymbol{R}} = \underbrace{\begin{pmatrix} H_1 & H_2 \\ H_2^* & -H_1^* \end{pmatrix}}_{\boldsymbol{H}} \cdot \underbrace{\begin{pmatrix} S_1 \\ S_2 \end{pmatrix}}_{\boldsymbol{S}} + \underbrace{\begin{pmatrix} N_1 \\ N_2 \end{pmatrix}}_{\boldsymbol{N}}, \qquad (4)$$

where $\boldsymbol{H}$ is a combined channel value/transmission scheme matrix. With this notation the soft demodulation is [5]

$$\Lambda(b_i|\boldsymbol{R}) = \ln\left(\frac{\sum_{\boldsymbol{S}\in\mathcal{S}_i^{(0)}} e^{\left(-\frac{1}{2\sigma_N^2}\|\boldsymbol{R}-\boldsymbol{HS}\|^2\right)}}{\sum_{\boldsymbol{S}\in\mathcal{S}_i^{(1)}} e^{\left(-\frac{1}{2\sigma_N^2}\|\boldsymbol{R}-\boldsymbol{HS}\|^2\right)}}\right), \qquad (5)$$

where $\mathcal{S}_i^{(0)}$ and $\mathcal{S}_i^{(1)}$ is the set of transmitted signals $\boldsymbol{S}$ with $b_i = 0$ and $b_i = 1$, respectively.

In Fig. 7, the BERs of the one antenna and two antennas transmission schemes with CDD and the Alamouti scheme for BPSK modulation are shown. Additionally, the BER of the CDD with three antennas is given. The channel has typical flat fading characteristics. For uncoded transmission there are large gains in using the Alamouti scheme compared to the two antennas transmit CDD (for instance 6 dB at a BER of 10^{-2}). But with channel coding there remains only a slight difference of approximately 0.7 dB, where we used a memory two convolutional code with generator polynomial $\boldsymbol{G}(D) = (1 + D^2, 1 + D + D^2)$. And the scheme can be easily extended to more then two antennas with additional gains of about 1.8 dB at a BER of 10^{-3} in the three antennas case (here with $\delta_{cyc,1} = 0$, $\delta_{cyc,2} = 21$ and $\delta_{cyc,2} = 42$ in samples). Thus, even with the best choice of the cyclic shift ($\Delta_{opt}^{BPSK} = 32$) we cannot achieve the performance of the Alamouti scheme in the coded transmission, but on the other hand there is no additional effort in the receiver for the combining respective for the soft demodulation of a vector. Because the complexity of this demodulation increases quadratically with the modulation alphabet.

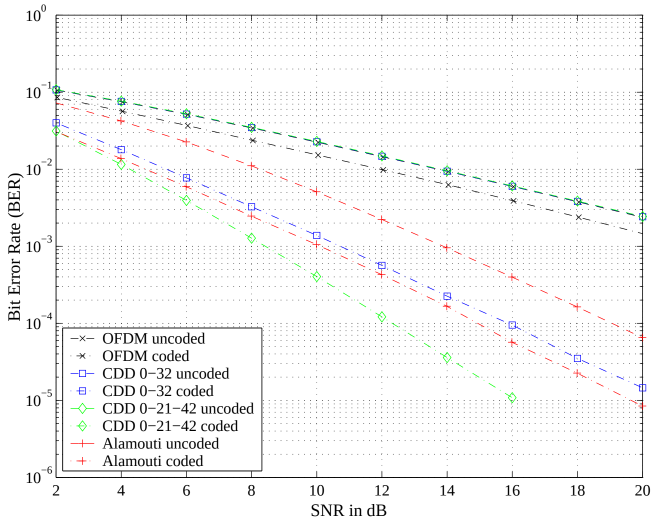

Fig. 7. Comparison of CDD and the Alamouti Scheme for BPSK transmission

In Fig. 8, for QPSK modulation the BERs of the Alamouti scheme and CDD are shown. For uncoded transmission the BERs of the one antenna and two antennas CDD are the same. Due to the higher order modulation scheme we even gain about 9 dB at a BER of 10^{-2} from the Alamouti scheme when transmitting uncoded. But for coded transmission we end up at 0.7 dB over the whole SNR range. It must be pointed out that the optimum cyclic shift is now $\Delta_{cyc}^{QPSK} = 16$.

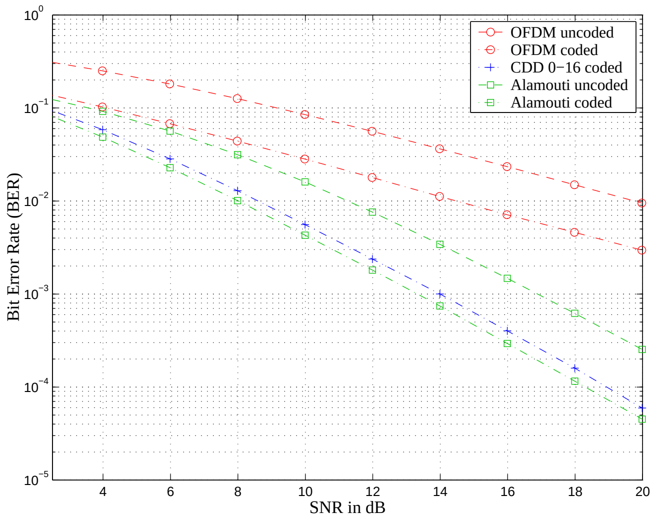

Fig. 8. Comparison of CDD and the Alamouti Scheme for QPSK transmission

V. Conclusions

In this paper we have investigated the performance of a simple multiple antenna transmission scheme – namely Cyclic Delay Diversity (CDD). This diversity method works only in combination with channel coding when using the same receiver structure as in the one antenna case. But that is no restriction in our opinion, because in nowadays communication systems always channel coding is used. We have analyzed how the cyclic shift has to be chosen in dependency of the number of antennas and the modulation alphabet (for PSK modulation) to obtain best results.

Simulation results have shown that we can nearly reach the performance of the well-known Alamouti scheme that achieves full spatial diversity. But in contrast to this without any combining scheme in the receiver and thus, without additional complexity. So, it provides a high measure of flexibility, since without any signaling the transmitter can switch between one antenna and multiple antenna transmission.

References

[1] S.M. Alamouti, *A Simple Transmit Diversity Technique for Wireless Communications*, IEEE Journal on Selected Areas in Communications, vol. 16, no 8, October 1998.

[2] I. Gaspard and J. Winter, *A New Space Diversity Scheme for DVB-T*, 5th International OFDM-Workshop (InOWo), Hamburg, 2000.

[3] A. Damman and S. Kaiser, *Low Complex Standard Conformable Antenna Diversity Techniques for OFDM Systems and its Application to the DVB-T System*, 4th International ITG Conference on Source and Channel Coding, Berlin, 2002.

[4] A. Damman, P. Lusina, and M. Bossert, *On the Equivalence of Space-Time Block Coding with Multipath Propagation and/or Cyclic Delay Diversity in OFDM*, IEEE European Wireless, Florence, Italy, 2002.

[5] H. Schulze, *Simple Transmit Diversity for a Convolutionally Coded Multicarrier QAM System*, 6th International OFDM-Workshop (InOWo), Hamburg, 2001.

[6] M. Gidlund, *Enhancement of HIPERLAN/2 Systems using Space-Time Coding*, IEEE European Wireless, Florence, Italy, 2002.